How To Calculate Required Material Thickness For Making Graded Shielding
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[Edit - 12 Dec 2013, broken link to NDT Resource Center fixed.]
[Edit - 14 Dec 2013, information on Pewter added in "Calculations Summary"]

In GammaSpec message #16270 George described the graded shielding method for knocking down X-rays emanating from shielding used around sources/sensors. He said 1/16 inch Cadmium makes a big improvement, but we can substitute Tin instead. Cadmium has become very difficult for us to find so we need to make this substitution. I have been looking for an explanation of how to calculate shielding thickness for various energy levels to see how much variation in shielding thickness we might find using different materials. The following websites answered a lot of questions I had and cleared up some confusion. I thought I would pass this along for those who are interested. In addition, I show how to calculate the thickness values of tin, copper and Polymethyl methacrylate plastic (PMMA, Acrylic) to reduce the 75/85 KeV Lead X-rays to less than 10 percent of the initial radiation.

http://www.sprawls.org/ppmi2/RADPEN/ gives a good introduction to radiation penetration and describes the Half Value Layer (HVL), the thickness of an absorber penetrated by one half of the incident radiation.

The next one goes into more detail on the theory behind calculating the HVL:
http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/radmatinteraction.htm
This opens at "Interaction Between Penetrating Radiation and Matter". Clicking the "Next" button goes to "Transmitted Intensity and Linear Attenuation Coefficient", followed by "Half-Value Layer".

What follows is my summary and interpretation of key points from the previous web sites. If I have misstated or misinterpreted anything, please let me know.

Linear Attenuation Coefficient

The linear attenuation coefficient **μ** (pronounced "mu") describes the fraction of a beam of x-rays or gamma rays that is absorbed or scattered per unit thickness of the absorber. Linear attenuation coefficients can sometimes be found but the data is frequently given in terms of the mass attenuation coefficient. Mass attenuation coefficient is defined as μ divided by the material's density **ρ** (pronounced "rho"), μ/ρ. It has units of cm^2/gm .

To convert a mass attenuation coefficient (μ/ρ) to a linear attenuation coefficient (μ), simply multiply it by the density (ρ) of the material.

$$\mu=(\frac{\mu}{\rho})*\rho$$

Half-Value Layer

The thickness of any given material where 50% of the incident energy has been attenuated is known as the half-value layer (HVL). The HVL is expressed in units of distance (mm or cm). Like the attenuation coefficient, it is photon energy dependent. Increasing the penetrating energy of a stream of photons will result in an increase in a material's HVL.

The HVL is inversely proportional to the attenuation coefficient, μ, and is an exponential function. If an incident energy of 1 and a transmitted energy is 0.5 is plugged into the HVL equation it can be shown that the HVL multiplied by μ must equal 0.693. This is because the number 0.693 is the exponent value that gives a value of 0.5 for transmitted energy

This leads to: $HVL = \frac{0.693}{\mu}$

Tables of μ and ρ for many materials, including the ones we are most interested in, are found at a NIST website at:
http://www.nist.gov/pml/data/xraycoef/index.cfm
along with some explanations of procedures in calculating them. Bookmark this site - it has all the data you need to calculate shielding characteristics of many elements and a number of compounds. Better yet, save the web page files to your local computer so you can use the data offline. You can download just the individual elements and compounds you are interested in.

Compare HVL of Cadmium and Tin

The X-ray energies from Lead are 75 and 85 KeV. The tables at the NIST website give us values for μ at 80 and 100 KeV. The slope of the curve there is very steep, so a small difference in energy makes a large difference in HVL. We can take the conservative approach and calculate HVL for 100 KeV, which should get us a little better performance than predicted.

Penetration = $(0.5)^N$
where N = number of HVLs

Cadmium
100 KeV HVL of Cadmium Z = 48 and ρ = 8.650
μ/ρ = 1.524, μ = (1.524)(8.050) = 12.2682
HVL = (0.693) / (12.2682) = 0.0565 cm = 0.0222 in
Penetration: 3 HVLs = 0.0666 in, resulting in 12.5% passing
Penetration: 4 HVLs = 0.0888 in, resulting in 6.25% passing - better than 90% reduction

Tin
100 KeV HVL of Tin Z = 50 and ρ = 7.310
μ/ρ = 1.676, μ = (1.676)(7.310) = 12.2516
HVL = (0.693) / (12.2516) = 0.0566 cm = 0.0223 in
Penetration: 3 HVLs = 0.0669 in
Penetration: 4 HVLs = 0.0892 in

Comparison

HVL @ 100 KeV	Cadmium	Tin
3	0.0666 in	0.0669 in
4	0.0888 in	0.0892 in

So Tin is very close to Cadmium for shielding at 100 KeV - certainly by the time the numbers are rounded up to match commercially available sheet stock, there is no practical difference. Both Cd and Sn have melting points lower that of Pb, so neither can be put into place prior to casting lead without creating an alloy at the junction rather than a boundary of two discrete elements.

Next Layer - Copper

Tin has XRF energies of 25 and 28 KeV, just slightly higher than Cadmium's 23 and 26 KeV. We can still shield them with Copper, which has a ρ of 8.960. We can use Copper's μ for 30 KeV from NIST Table 3.

$\mu/\rho = 10.92$ at 30KeV, $\mu = (10.92)\ (8.960) = 97.8432$
HVL = (0.693) / (97.8432) = 0.0071 cm = 0.0028 in
3 HVLs = 0.0084 in
4 HVLs = 0.0112

Skip the Tin?

To do it all with Copper (no Tin)
Cu $\mu/\rho = 0.4584$ at 100 KeV, $\mu = (0.4584)\ (8.960) = 4.1073$
HVL = (0.693) / (4.1073) = 0.1687 cm = 0.0664
3 HVLs = 0.1992 in
4 HVLs = 0.2656 in

So at 100 KeV, it would require a thickness of over 1/4 inch of Copper to accomplish what we achieved with a combination of about 0.09 inch Tin plus 0.02 in Copper.

Last Layer - Acrylic Plastic (PMMA)

Copper XRF energies are given as 8 and 9 KeV, so we can add a layer of Acrylic plastic to block them. Fortunately, we can obtain Acrylic's ρ and μ/ρ from NIST Table 2 (Material constants and composition for compounds and mixtures) and Table 4 ([Data] compounds and mixtures).

Acrylic $\rho=1.190$, $\mu/\rho = 3.357$ at 10 KeV, $\mu = (3.357)\ (1.190) = 3.9948$
HVL = (0.693) / (3.9948) = 0.1735 cm = 0.06831 in
3 HVLs = 0.2049 in
4 HVLs = 0.2732 in

This time it looks like we might need to round 0.2732 in down to 0.25 to get a reasonable standard thickness. Let's take a look at what would happen if we used an energy of 8 KeV (the lower value for Copper) instead of 10 KeV. We could also interpolate between 8 and 10 KeV, but let's use the lower value.

Acrylic $\rho=1.190$, $\mu/\rho = 6.494$ at 8 KeV, $\mu = (6.494)\ (1.190) = 7.7279$
HVL = (0.693) / (7.729) = 0.0897 cm = 0.0353 in
3 HVLs = 0.1059 in
4 HVLs = 0.1412 in

We can see how steep the characteristic curve is here - dropping the energy from 10 KeV to 8 KeV cut the PMMA HVL shielding almost in half. Since Copper's two X-rays are 8 and 9 KeV, our choice of 0.25 in PMMA should give us about 4 HVLs at 9 KeV.

Calculations Summary

Going through the math shows us that we can gain reasonable shielding of the 75/85 KeV Lead X-rays without going overboard on materials. We should be able to cut the XRF radiation by more than 90% by lining our lead castle with (rounding off) 0.09 in Tin, 0.015 Copper and .25 Acrylic.

Currently, there seems to be a problem finding pure tin in sheet form. Tin ingots are readily available on eBay, but the hobby-shop "Tin Sheet" sold there is actually steel with a thin tin plating. The same is true for hobby-shop copper sheet - it is copper plated steel.

However, Wally94087 of the list pointed out a source of Pewter with a known analysis of 92% tin, 7.5% antimony, and 0.5% copper. Antimony (Sb) is adjacent to Sn in the periodic table, and has XRF energies of 26.36 and 29.72 KeV compared to Sn's 25.27 and 28.48 KeV, so they are comparable. The NIST site says their data for mixtures and compounds were obtained by adding the individual contributions of each constituent according to their fractions by weight, so similar numbers could be calculated for Pewter if anybody really wanted to. Based on all that, it appears Pewter should be an acceptable substitute since it does not contain elements that would introduce new X-rays outside of what we expected from Cd or Sn and the K-edge for Sb is, like Cd and Sn, well away from our 80+/- KeV area of interest. It is relatively inexpensive and comes in sheet form that would make it easily incorporated into the shield.

The Contenti Company at
http://www.contenti.com
currently offers 6 in. by 12 in. Pewter sheet in thicknesses starting at 0.0312 in. (0.8 mm). They have several thickness choices within our range of interest so the shield could be tailored to whatever number of HVLs is desired. Since their Pewter is 92% tin, using μ and ρ for tin should be sufficiently accurate for our purposes. They also have the same Pewter composition in casting ingots if casting a floor is desired. In addition, they have high purity 12 in. by 12 in. dead soft copper (99.9%) sheet in thicknesses suitable for the copper layer of our shield.

This has been my interpretation of key points from the previous web sites and several informative posts by list members Wally94087 and Randall Buck. If I have misstated or misinterpreted anything, please let me know.

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